DELTA MATH SCIENCE PARTNERSHIP INITIATIVE

M³ Summer Institutes

(Math, Middle School, MS Common Core)

Vocabulary and its Importance to Mathematics and Education

Basic Operational Keywords:

Addition is to increase, add to, gain, and many other words and phrases.

Subtraction is to *reduce*, make smaller, take away from, the difference in a comparison, or the missing amount needed to reach an objective amount.

Multiplication is to increase greatly, to add repeatedly, to complete a rectangle and measure its area, and has several ways of being viewed.

Division is to subtract repeatedly, to determine the measures of the sides of a rectangle of known area.

In order to understand the ideas behind any of these basic operations, one must understand the definitions, how to illustrate each one with an example, and the vocabulary required to explain why a true problem is solved this way.

Models to be used:

Set Model: a set model is used as illustration when someone is attempting to perform a basic operation on discrete objects such as cans of soda, pencils, coins (the amount of, not the value), puppies, etc.

Compare this to a measurement model.

Measurement Model: a measurement model is used for illustration when someone is attempting to perform basic operations on continuous amounts such as length, volume (ounces, cubic feet, etc.), value of money, time, etc.

These are the only two models used. All situations are reducible to one of these two models, and in some cases, either model might be used depending on the point of view of the reader.

When a student is asked to read a problem, the words may be understandable but not the meaning once taken as a whole. Mathematics lends itself to one of two models as outlined above, and then next an approach to lead to understanding. Approaches are operation specific whereas models are not. Hence, with each operation, the student must understand various approaches and be careful to not confuse them.

Addition has only two approaches named after the two models: *set* and *measurement*.

The anatomy of an addition problem is as follows:

12 + 34 = 46

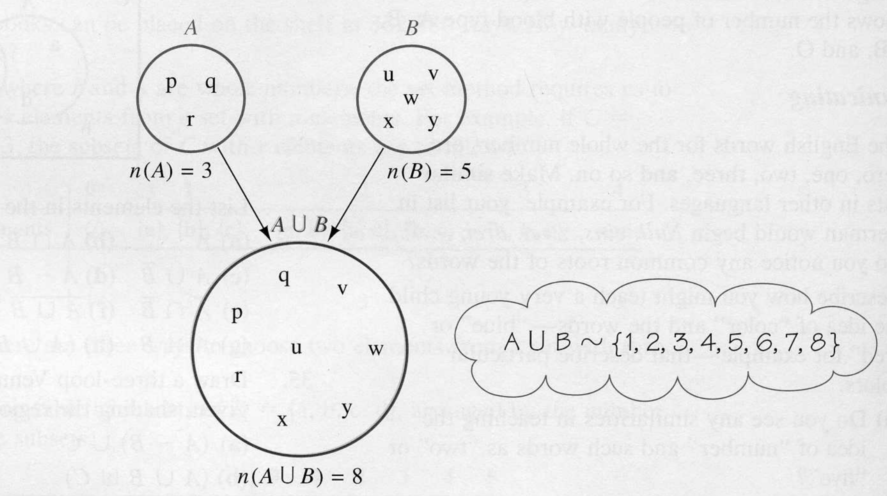
The 12 and 34 are called **addends** and the result or answer to the problem is the **sum**.

Here is the general definition.

DEFINITION The Addition of Whole Numbers

Let a and b be any two whole numbers. If A and B are any two disjoint sets for which $a = n(A)$ and $b = n(B)$, then the **sum of a and b,** written $a + b$, is given by $a + b = n(A \cup B)$.

$n(A \cup B) = 8$, we have shown that $3 + 5 = 8$.



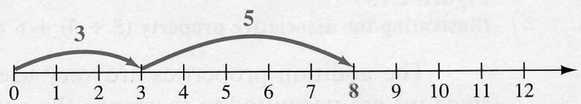


Figure 2.14
Illustrating 3 + 5 on the number line

With the following examples determine the approach and draw an illustration modeling each:

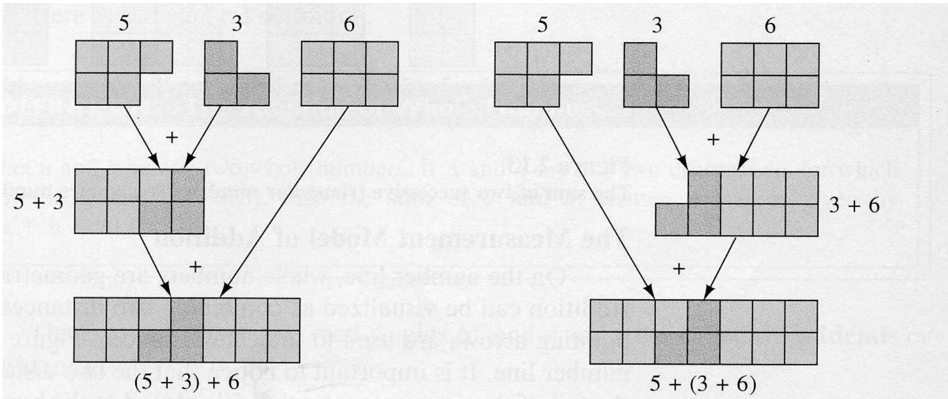
Scott walked out to the garden to pick ears of corn. Scott picked 5 ears on the first row and 6 ears on the second. How many ears of corn did Scott pick in all?

Scott walked 3 blocks to school and then 7 more blocks to the library after school. How many blocks did Scott walk in all?

With addition comes several properties that need to be understood.

THEOREM Properties of Whole Number Addition

Closure Property	If a and b are any two whole numbers, then $a + b$ is a unique whole number.
Commutative Property	If a and b are any two whole numbers, then $a + b = b + a$.
Associative Property	If a, b, and c are any three whole numbers, then $a + (b + c) = (a + b) + c$.
Additive Identity Property of Zero	If a is any whole number, then $a + 0 = 0 + a = a$.



Subtraction has three approaches to help explain the situation. The first is the usual **take-away** approach. The **take-away** approach begins with a known amount and a subset of this amount is removed (taken-away). The second approach is known as **comparison** and requires the student to know two amounts. These known amounts are compared to see which is greater and the difference is the answer. Comparison is easily seen to be different from the take-away approach as one has one total amount and the other has two known totals. The third and last of these is the **missing-addend** approach and, when read, seems to be an addition problem rather than a subtraction problem. The **missing–addend** approach is where an amount is given, known, and a future value is desired. The question asks how much more is needed to get this future value. This means that there are six different types of subtraction problems when the three approaches are mixed with the two types of models.

DEFINITION Subtraction of Whole Numbers

Let a and b be whole numbers. The **difference,** written $a - b$, is the unique whole number c such that $a = b + c$. That is, $a - b = c$ if, and only if, there is a whole number c such that $a = b + c$.

The anatomy of a subtraction problem is as follows:

102 - 34 = 68

102 is called the **minuend**, 34 is the **subtrahend** and the **difference** is 68.

With the following examples determine the approach and draw an illustration modeling each:

Jane wants to purchase a new stereo for her car. She is interested in buying a stereo that costs $150, however, she has only saved $65. How much more money does she need to save to make the purchase?

LaJarvis was walking across his front yard carrying a glass of tea when he stumbled. He spilt 8 ounces of the 23 ounces of tea in his glass. How much tea is left in the glass?

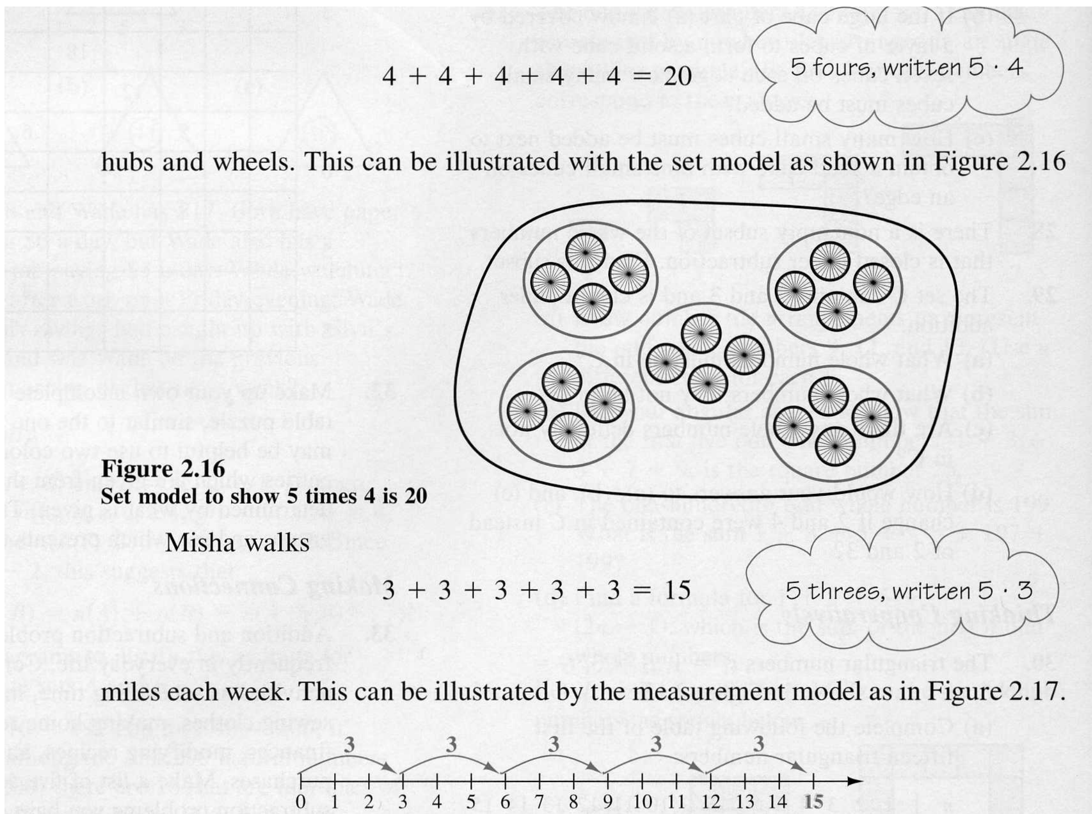
LaJarvis and his sister, Tammy, have a disagreement over who can jump the greatest distance. Both run 10 feet to a line drawn on the ground and then jumps. LaJarvis jumps 138 centimeters and Tammy jumps 154 centimeters. Who jumped the greater distance and how much farther did they jump?

Lamar picked 15 pears for an after dinner desert. After cutting them open, it is found that 7 are spoiled. How many of the pears are good to eat?

Jaunita runs a successful chicken egg farm. She has four eggs in a carton that holds one dozen eggs. How many more eggs does she need to collect to fill the carton?

Hans has collected 6 red pens and 4 blue pens. Of which color does he have the most and how many more of them does he have?

Multiplication has three approaches: **repeated addition**, **rectangular array,** and **cross-product**. With these come two different definitions based on the approach being used.



The anatomy of a multiplication problem is as follows:

17 * 6 = 102

17 and 6 are called factors and the product is 102, however these words hide some interest problems. Using older language, 17 is the **multiplier** and 6 is the **multiplicand**. These terms have fallen into disuse but gave a fuller explanation of the definition of multiplication than is currently observed.

DEFINITION Multiplication of Whole Numbers as Repeated Additions

Let a and b be any two whole numbers. Then the **product** of a and b, written $a \cdot b$, is defined by:

$$a \cdot b = \underbrace{b + b + \cdots + b}_{a \text{ addends}} \quad \text{when} \quad a \neq 0;$$

and by:

$$0 \cdot b = 0.$$

With the following examples, determine the approach and draw an illustration modeling each:

Marcus buys four 6-packs of canned soda for a party. How many cans of soda did he buy?

Tameka plants a small garden with five rows of equal length each containing eight tomato plants. How many tomato plants did she plant?

Scott decides to make a sandwich and finds that he has two types of bread in the house. He also has ham and turkey meat along with a tomato. Using just these ingredients how many different sandwiches can he make?

PROPERTIES Whole Number Multiplication

Closure Property If a and b are any two whole numbers, then $a \cdot b$ is a unique whole number.

Commutative Property If a and b are any two whole numbers, then $a \cdot b = b \cdot a$

Associative Property If a, b, and c are any three whole numbers, then $a \cdot (b \cdot c) = (a \cdot b) \cdot (c)$

Multiplicative Identity Property of One The number 1 is the unique whole number for which $b \cdot 1 = 1 \cdot b = b$ holds for all whole numbers b.

Multiplication by Zero Property For all whole numbers b, $0 \cdot b = b \cdot 0 = 0$.

Distributive Property of Multiplication over Addition If a, b, and c are any three whole numbers, then $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$ and $(a + b) \cdot c = (a \cdot c) + (b \cdot c)$.

Division has three approaches: **repeated subtraction**, **partition,** and **missing-factor**.

The anatomy of a division problem is as follows:

58 divided by 8 = 7 remainder 2.

Fifty-eight is the **dividend**, 8 is the **divisor** with the **quotient** being 7. The **remainder** is 2, and as this illustration or example is whole number division, a remainder quite often occurs. If the result is given as $7\frac{2}{5}$, the whole amount is called the quotient.

Division of Whole Numbers

There are three conceptual models for the division $a \div b$ of a whole number a by a nonzero whole number b: the **repeated-subtraction** model, the **partition** model and the **missing-factor** model.

The Repeated-Subtraction Model of Division

Ms. Rislov has 28 students in her class, that she wishes to divide into cooperative learning groups of 4 students per group. If each group requires a set of Cuisenaire® rods, how many sets of rods must Ms. Rislov have available? The answer, 7, is pictured in Figure 2.22, and is obtained by counting how many times groups of 4 can be formed, starting with 28. Thus, $28 \div 4 = 7$. The repeated subtraction model can be realized easily with physical objects; the process is called **division by grouping.**

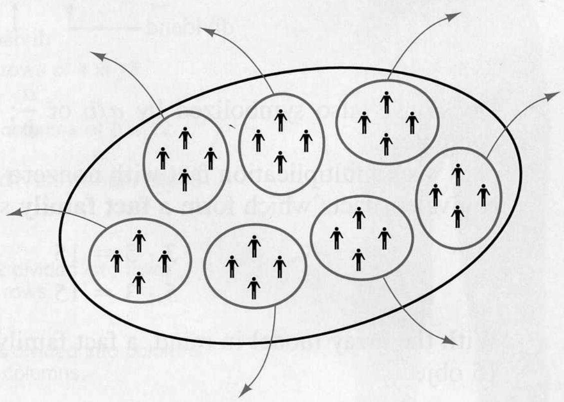


Figure 2.22
Division as repeated subtraction: $28 \div 4 = 7$ because seven 4s can be subtracted from 28.

The Partition Model of Division

When Ms. Rislov checked her supply cupboard, she discovered she had only 4 sets of Cuisenaire® rods to use with the 28 students in her class. How many students must she assign to each set of rods? The answer, 7 students in each group, is depicted in Figure 2.23. The partition model is also realized easily with physical objects, in which case the process is called **division by sharing.**

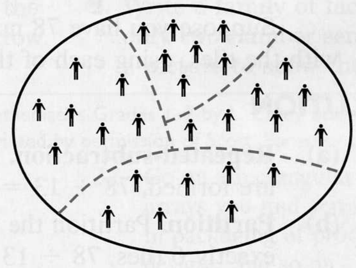


Figure 2.23
Division as a partition: $28 \div 4 = 7$ because when 28 objects are partitioned into 4 equal sized groups, there are 7 objects in each group.

With the following examples determine the approach and draw an illustration modeling each.

Molly is sharing some sweets with her friends. She has 42 sweets and, counting her, there are seven people in total. How many sweets does each person receive?

Sandra is pouring tea for people to have at lunch. Each glass will hold 8 ounces and her pitcher contains 96 ounces. How many glasses can she fill?

Harrison is the cake chef for a large restaurant and needs 96 eggs for all the cakes he is to bake. He is buying eggs in bunches of 6. How many bunches of eggs does he need to purchase?

Words to be avoided:

Cancel – no such mathematical operation exists. This word is used in place of phrases such as "add to zero", "subtract to zero", or "divide to one". Force students to use the correct mathematical terms so that students begin to understand what they are doing beyond marking off terms or factors.

Reduce – "To reduce" is to make smaller with subtraction whereas fractions undergo a "division to one" process. Fractions do not undergo the mathematical operation of reduction.